# Empirical Model for Forecasting Exchange Rate Dynamics: the GO-GARCH Approach

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*The study aimed at determining a set of superior generalized orthogonal-GARCH (GO-GARCH) models for forecasting time-varying conditional correlations and variances of five foreign exchange rates vis-à-vis the Nigerian Naira. Daily data covering the period 02/01/2009 to 19/03/2015 was used, and four estimators of the GO-GARCH model were considered for fitting the models. Forecast performance tests were conducted using the Diebold-Mariano (DM) and the model confidence set (MCS) tests procedures. The DM test indicates preference for the GO-GARCH model estimated with nonlinear least squares (NLS) estimator – denoted as GOGARCH-NLS, while the MCS test determined a set of superior models (SSM) which comprised of GO-GARCH-NLS and GOGARH model estimated by the method-of-moment, denoted as GO-GARCH-MM. These models were deemed best and adequate for forecasting of the five exchange rate dynamics.*

**Keywords**: MGARCH, GO-GARCH, conditional heteroscedasticity, volatility, time-varying correlation

**JEL Classification:** C32, C53, F31

## 1.0 Introduction

The foreign exchange market is very crucial in international trade, as many world economies are directly or indirectly linked through export and import trades. Nigeria, a growing third world economy, is not left out in this loop of inter-linkages as most of its raw material and machinery needed for industrial production are usually imported. Thus, the Nigerian foreign exchange market plays a vital role in this regard. Sudden and unexpected changes in the dynamics of exchange rates if not adequately monitored could lead to economic crises such as the Mexican Peso crisis, Euro zone currency and sovereign debt crisis, and the South East Asian crisis. Regular monitoring of the dynamics (especially, exchange rate volatilities and co-volatilities) that characterize the foreign exchange market will ensure a stable economy and further boosting of investors' confidence.

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In order to ensure economic stability in the country, the Central Bank of Nigeria has intervened actively in maintaining order by issuing exchange rate policies in the Nigerian foreign exchange market. The market had undergone four regimes of policy regulations between 1959 and 2010[2]. Within this period, the floating exchange rate system was introduced. Unlike the fixed exchange rate system, the floating exchange rate system introduces a lot of randomness in the rates' dynamics thereby making it difficult for precise forecasting of future values.

Several studies on the Naira exchange rates have been conducted on the Nigerian foreign exchange market, especially the rates of the Naira vis-à-vis foreign currencies such as the US dollar, British Pound Sterling, Euro, etc. Some of these studies are concerned with the investigation of relationships between exchange rates and macroeconomic variables. Recent studies include: Adamu (2005), Mordi (2006), Yaya and Shittu(2010), Mbutor (2010), Kelilume and Salami (2012), Usman and Adejare (2013) and Fapetu and Oloyede (2014). While others examined modeling and forecasting of exchange rates volatilities in the time series context using univariate *autoregressive conditional heteroscedasticity* (ARCH) model of Engle (1982) and the *generalized ARCH* (GARCH) model of Bollerslev (1986), as well as various extensions of these models[3]. Though, these univariate models are well known for their ability in capturing adequately volatility and stylized facts of univariate economic and financial time series, they are however not adequate when interest is bothered on modeling volatilities and co-volatilities of a system of time series.

In finance, the knowledge of joint movements of a set of assets (in terms of conditional variances and conditional covariances) is an essential requirement for efficient management and monitoring of financial portfolios. Forecasting of *Value-at-Risk* (VaR) thresholds also require the knowledge, while hedging and asset specialization strategies can be determined with the knowledge of conditional cross-correlations amongst assets (Caporin and McAleer, 2009). The *multivariate GARCH* (MGARCH) models provide avenues where these financial tasks can be easily carried out. They are capable in modeling second-order moments and inter-linkages inherent in a multivariate set of time series. Since the aim of this study is to forecast the dynamics (i.e. conditional

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[2]The four exchange rate regime include: the Fixed Parity (1959-1985), the Second-Tier Forex Market (SFEM, 1986 – 1994), the Autonomous Forex Market (AFEM, 1995 – 1999), and the Inter-Bank Forex Market (IFEM, 2000 – 2010).

[3]Reviews of the studies are presented in the literature review section.

volatilities and conditional correlations) of a system of five foreign exchange rates vis-à-vis the Nigerian Naira, we resort to a class of the MGARCH models called the *generalized orthogonal* GARCH (GO-GARCH) models. This class of models is based on the assumption that the co-movements of financial returns are driven by a small number of common underlying variables called *factors*.

The rest of the paper is structured as follows: Section 2 presents literature review; Section 3 presents structures of the general MGARCH and the GO-GARCH models, and forecast performance comparisons; Section 4 presents empirical data analysis, results and discussion; while Section 5 concludes the paper.

## 2.0 Literature review

Modeling and forecasting of exchange rate dynamics has many applications in economics, finance and other investment related fields. For investors and policy makers, the importance of exchange rate volatility in planning and decision making cannot be ruled out. The univariate ARCH/GARCH models have been quoted to record tremendous successes in modeling second-order moments of financial and economic time series. These models have found applications in modeling and forecasting of volatilities in financial markets such as stock exchanges, bond markets as well as the foreign exchange markets. Earlier studies which have applied the univariate ARCH/GARCH schemes in the modeling of second-order moments or volatilities of foreign exchange rates are those of Bollerslev (1987), Hsieh (1989) and Andersen and Bollerslev (1998). Some of the recent studies that have applied the schemes successfully in the modeling of exchange rate volatilities include: Balaban (2004), Cheong-Vee et al. (2011), Alam and Rahman (2012), and Xu et al. (2012).

Balaban (2004) compare performances of symmetric and asymmetric GARCH (1, 1), GJR-GARCH (1, 1) and the EGARCH (1, 1) models in forecasting volatility of the US Dollar/Deutsche Mark returns. The results of their study show that the EGARCH and the GARCH models relatively performed better than the GJR-GARCH model. Cheong-Vee et al. (2011) evaluated volatility forecasts of the US Dollar against the Mauritian Rupee exchange rate using GARCH (1, 1) models with GED and Student's t error distributions. Daily data spanning the period 30/06/2003 and 31/03/2008 was used in the study. Results of the study show that the GARCH (1, 1) model with GED errors slightly outperformed the other models. Alam and Rahman (2012) examined the BDT/US Dollar exchange rate volatility using GARCH-type models with daily data for the period 03/07/2006 to 30/04/2012. Their findings show the current volatility is significantly affected by past volatilities. Xu et al. (2012) in their study compare the performance of Realized GARCH model with GARCH (1, 1) and IGARCH (1, 1) models using 10-minute intra-day closing prices of spot rates of eight exchange rates (AUD/USD, EUR/GBP, EUR/JPY, EUR/USD, GBP/USD, CAD/USD, CHF/USD & JPY/USD) that span the period 04/08/2003 and 03/08/2010. The results of their study show that the Log-Linear Realized GARCH model outperformed the other models both in the in-sample and out-sample data sets. Considering weekly returns data, the GARCH (1, 1) model outperformed the other models in the out-sample data. A noteworthy shortcoming associated with the modeling method adopted by these authors is that the joint movements and inter-linkages amongst the exchange rates were not accounted for. Knowledge of these co-movements will be vital for investment planning. An alternative method is to model the rates simultaneously using a multivariate volatility model.

In Nigerian, the univariate ARCH/GARCH models have equally been used by some researchers in analyzing the Naira rates vis-à-vis other foreign currencies. Such studies include those of Olowe (2009), Awogbemi and Alagbe (2011), Adeoye and Atanda (2012), and Bala and Asemota (2013) to mention a few. Olowe (2009) examined the log-returns volatility of the average Naira/Dollar exchange rates using GARCH (1, 1), GJR-GARCH (1, 1), EGARCH (1, 1), APARCH (1, 1), IGARCH (1, 1) and TS-GARCH (1, 1) models using monthly data spanning the period January, 1970 to December, 2007. The study assessed the effects of asymmetry and volatility persistence, as well as the impact of deregulation of the Nigerian foreign exchange market. The results of study show strong evidence of volatility persistence during the sampled period as well as significant asymmetric effects on the volatility process. Awogbemi and Alagbe (2011) examined volatility of the Naira/US Dollar and the Naira/Pound Sterling exchange rates using separate GARCH (1, 1) models and monthly data that spanned the period 2006 and 2010. Their finding indicates existence of volatility persistence in the exchange rate returns. As an observation, the separate use of univariate GARCH models in modeling the two exchange rates however fails to capture their joint movements over time. Knowledge of these co-movements could facilitate proper planning and decision making by importers, exporters and investors who embark on foreign businesses. This drawback as seen in the study could easily be solved by simultaneously modeling the two rates via a bivariate GARCH model. Adeoye and Atanda (2012) examined consistency, persistency and the degree of volatility in the Naira/US Dollar exchange rates using monthly data for the period 1986 through 2008. The ARCH/GARCH models were used in assessing the severity of volatility in the nominal and real exchange rates. Their results also confirm the existence of volatility persistence in both the nominal and real exchange rates. Bala and Asemota (2013) examined volatility of three exchange rates (US Dollar, Pound Sterling and Euro) vis-à-vis the Naira using GARCH models and monthly data that spanned the period January 1985 and July 2011. The study compared performance of variants of the GARCH models with and without the incorporation of exogenous breaks in model estimation. The findings of their study show that performance of the models improved by incorporating volatility breaks in the estimated models. Furthermore, all of the asymmetric models fitted in the study reject existence of leverage effects in the volatility processes. As an alternative, a trivariate MGARCH model could have been used to simultaneously capture the second-order moments as well as the inter-links inherent in the system of exchange rates. Musa et al. (2014) also examined volatility of the Naira/US Dollar rates using symmetric GARCH, GJR-GARCH, TGARCH and TS-GARCH models with daily data covering the period June 2000 to July 2011. Statistically significant asymmetric effects were reported from the fitted GJR-GARCH and TGARCH models. Also in terms of forecast performance, the TGARCH model was reported to provide better forecasts than the other models.

Each of the studies cited above used univariate GARCH-type models in the modeling of volatilities of exchange rates. But when interest is bothered on modeling of the joint evolution of a system of time series, these models become inadequate and needed to be extended to a multivariate framework. The MGARCH framework provides avenues for modeling second-order moments and interlinks existing in a system of time series. Bauwens et al. (2006) and Silvennoinen and Terasvirta (2008) provide reviews on the MGARCH models.

The MGARCH models have been applied successfully in the modeling and forecasting of volatilities and dynamic correlations in foreign exchange markets. Some of the recent exchange rate studies in the MGARCH context include: Bollerslev (1990), Hartman and Sedlak (2013), Patnaik (2013) and Pelinescu (2014). Bollerslev (1990) introduced the CCC-GARCH model. This model was the first model used in examining exchange rate volatilities of five exchange rates in the multivariate context with the assumption of constant conditional correlations. Patnaik (2013) investigated exchange rate volatilities of the Indian Rupee vis-à-vis the US dollar, Pound Sterling, Euro and the Japanese Yen using daily data spanning the period 5/4/2010 and 18/7/2011. Evidence of volatility spillovers among the rates was reported, even though they are not statistically significant. The study further revealed that volatility spillovers seem not to pose serious problems in the Indian foreign exchange market. In Pelinescu (2014), the diagonal BEKK-GARCH models with scalar restriction was used in analyzing the Romanian RON/Euro, the US Dollar/Euro, the Polish Zloty/Euro and the Czech Republic Koruna/Euro exchange rates. Strong asymmetry was discovered in the rates, it was also discovered that the returns were highly correlated with volatilities. In similar studies, Zahnd (2002), Andersen et al. (2003), Pesaran et al. (2008), Hartman and Sedlak (2013) have also applied MGARCH models in the analyses of exchange rate volatilities.

In Nigeria, and to the best of our knowledge, no study on the Naira exchange rate volatility co-movements using the MGARCH framework had been conducted or was found as at the time of this review. As such, the study aims at modeling the joint evolution of the Naira exchange rates vis-à-vis five foreign currencies using a variant of the MGARCH models (the GO-GARCH model), with the intention of forecasting the exchange rates volatilities and dynamic conditional correlations.

## 3.0 Statistical preliminaries

### 3.1 The general MGARCH model

Let $\{\boldsymbol{y}_t\}$ denote a covariance stationary $N \times 1$ dimensional vector stochastic process with properties $E(\boldsymbol{y}_t|\boldsymbol{\mathcal{F}}_{t-1}) \equiv \boldsymbol{\mu}_t(\boldsymbol{\theta})$ and $var(\boldsymbol{y}_t|\boldsymbol{\mathcal{F}}_{t-1}) \equiv \boldsymbol{H}_t$, where the sigma field generated by the vector process denoted by $\boldsymbol{\mathcal{F}}_{t-1}$ is the information set of the process up to time $t-1$, and $N$ is the number of time series in $\boldsymbol{y}_t$. Bauwens et al. (2006) defines $\{\boldsymbol{y}_t\}$ as:

$$\boldsymbol{y}_t = \boldsymbol{\mu}_t(\boldsymbol{\theta}) + \boldsymbol{\varepsilon}_t \tag{1}$$

where $\boldsymbol{\mu}_t(\boldsymbol{\theta})$ is a conditional mean vector dependent on a finite parameter vector $\boldsymbol{\theta}$.[4] The innovations vector process $\{\boldsymbol{\varepsilon}_t\}$ is heteroscedastic and is defined as:

$$\boldsymbol{\varepsilon}_t = \boldsymbol{H}_t^{1/2}(\boldsymbol{\theta})\boldsymbol{\eta}_t \tag{2}$$

[4]Note that both $\boldsymbol{\mu}_t$ and $\boldsymbol{H}_t$ depend on the parameter vector $\boldsymbol{\theta}$.

where $\boldsymbol{\eta}_t$ is a martingale-difference sequence with properties: $E(\boldsymbol{\eta}_t|\boldsymbol{\mathcal{F}}_{t-1}) \equiv \mathbf{0}$ and $var(\boldsymbol{\eta}_t|\boldsymbol{\mathcal{F}}_{t-1}) \equiv E(\boldsymbol{\eta}_t\boldsymbol{\eta}_t'|\boldsymbol{\mathcal{F}}_{t-1}) = \boldsymbol{I}_N$. The conditional covariance matrix of $\{\boldsymbol{y}_t\}$ denoted by $\boldsymbol{H}_t \equiv [h_{ij,t}]$ for $i,j = 1,2,\cdots,N$ (leaving out $\boldsymbol{\theta}$ notation for convinience) is given by:

$$
\begin{aligned}
var(\boldsymbol{y}_t|\boldsymbol{\mathcal{F}}_{t-1}) = var(\boldsymbol{\varepsilon}_t|\boldsymbol{\mathcal{F}}_{t-1}) &= \boldsymbol{H}_t^{1/2} var(\boldsymbol{\eta}_t|\boldsymbol{\mathcal{F}}_{t-1})\left(\boldsymbol{H}_t^{1/2}\right)' \\
&= \boldsymbol{H}_t
\end{aligned} \qquad (3)
$$

In general, Eq. (1) and Eq. (2) specify the structure of the MGARCH models. The interest in MGARCH is on the formulation and specification of the time-varying conditional covariance matrix, $\boldsymbol{H}_t$.

### 3.2 The generalized orthogonal-GARCH (GO-GARCH) model

Suppose that $\{\boldsymbol{\varepsilon}_t \equiv \boldsymbol{y}_t - \boldsymbol{\mu}_t, t = 1,2,\cdots,T\}$ denotes an N-dimensional innovation vector process, then the GO-GARCH scheme imposes a structure on the vector process $\{\boldsymbol{\varepsilon}_t\}$ through a linear invertible mapping matrix $\boldsymbol{W}$ and is defined as:

$$\boldsymbol{\varepsilon}_t = \boldsymbol{W}\boldsymbol{f}_t \qquad (4)$$

The linear map is an $N \times N$ parameter matrix that is constant over time, and $\boldsymbol{f}_t \equiv \left(f_{1,t}, f_{2,t}, \cdots, f_{N,t}\right)'$ denotes a vector of unobserved independent components or factors. The factors are defined by:

$$\boldsymbol{f}_t = \left(\boldsymbol{H}_t^f\right)^{1/2}\boldsymbol{\eta}_t \qquad (5)$$

where $\boldsymbol{H}_t^f \equiv E(\boldsymbol{f}_t\boldsymbol{f}_t'|\boldsymbol{\mathcal{F}}_{t-1}) = diag\left(h_{1,t}^f, h_{2,t}^f, \cdots, h_{N,t}^f\right)$ is an $N \times N$ diagonal matrix of conditional variances, and $\boldsymbol{\eta}_t \equiv \left(\eta_{1,t}, \eta_{2,t}, \cdots, \eta_{N,t}\right)'$. The random vector process $\{\boldsymbol{\eta}_t\}$ has the properties: $E(\boldsymbol{\eta}_t|\boldsymbol{\mathcal{F}}_{t-1}) \equiv \mathbf{0}$ and $E(\boldsymbol{\eta}_t\boldsymbol{\eta}_t'|\boldsymbol{\mathcal{F}}_{t-1}) \equiv \boldsymbol{I}_N$, since $\eta_{i,t}$ and $\eta_{j,t}$ are independent for every $i \neq j; i,j = 1,2,\cdots,N$. It implies that the conditional expectations: $E(\boldsymbol{f}_t|\boldsymbol{\mathcal{F}}_{t-1}) \equiv \mathbf{0}$ and $E(\boldsymbol{\varepsilon}_t|\boldsymbol{\mathcal{F}}_{t-1}) \equiv \mathbf{0}$. The unconditional distribution of the factors is characterized by: $E(\boldsymbol{f}_t) \equiv \mathbf{0}$ and $E(\boldsymbol{f}_t\boldsymbol{f}_t') \equiv \boldsymbol{I}_N$, which in turn implies that the unconditional distribution of the innovation vector process is also characterized by $E(\boldsymbol{\varepsilon}_t) = \mathbf{0}$ and $E(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t') = \boldsymbol{W}\boldsymbol{W}' = \boldsymbol{H}$. The conditional covariance matrix of the innovation vector $\boldsymbol{\varepsilon}_t$ is then defined as:

$$\boldsymbol{H}_t = E(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'|\boldsymbol{\mathcal{F}}_{t-1}) = \boldsymbol{W}\boldsymbol{H}_t^f\boldsymbol{W}' = \sum_{i=1}^{N} \boldsymbol{\omega}_{(k)}\boldsymbol{\omega}_{(k)}' h_{k,t}^f \qquad (6)$$

where $\boldsymbol{\omega}_{(k)}$, for $k = 1, 2, \cdots, N$ are the columns of $\boldsymbol{W}$ and $h^f_{k,t}$ are the diagonal elements of $\boldsymbol{H}^f_t$. The k^th^ factor or component GARCH ($p$, $q$) is defined as:

$$h^f_{k,t} = \omega_k + \sum_{i=1}^{q} \alpha_k f^2_{k,t-i} + \sum_{i=1}^{p} \beta_k h^f_{k,t-i}, \quad k = 1, 2, \cdots, N \tag{7}$$

And the conditional correlation matrix is obtained from Eq. (6) and is defined as:

$$\boldsymbol{R}_t = \boldsymbol{D}_t^{-1} \boldsymbol{H}_t \boldsymbol{D}_t^{-1} \tag{8}$$

where $\boldsymbol{D}_t^{-1} \equiv (\boldsymbol{H}_t \odot \boldsymbol{I}_N)^{1/2}$ and $\odot$ denotes the Hadamard product operator.

### 3.3 Estimators of the GO-GARCH model

***Maximum likelihood* (ML) *estimator***

The two-step ML estimator of van der Weide (2002) is obtained by maximizing the following multivariate Gaussian log-likelihood function:

$$L(\boldsymbol{\theta}, \boldsymbol{\alpha}, \boldsymbol{\beta}) = -\frac{1}{2} \sum_{t=1}^{T} N \log_e 2\pi + \log_e |\boldsymbol{W}_\theta \boldsymbol{W}'_\theta| + \log_e |\boldsymbol{H}_t| + \boldsymbol{f}'_t \boldsymbol{H}_t^{-1} \boldsymbol{f}_t \tag{9}$$

The first step identifies part of the linear mapping matrix, while the second step estimates the remaining part of the mapping matrix and the parameters of the component GARCH models.

***Nonlinear least squares* (NLS) *estimator***

The three-step NLS estimator of Boswijk and van der Weide (2006) is obtained by minimizing the nonlinear least squares criterion given by:

$$\begin{aligned} S(\boldsymbol{A}) &= \frac{1}{T} \sum_{t=1}^{T} tr([\boldsymbol{V}\boldsymbol{s}_t\boldsymbol{s}'_t\boldsymbol{V}' - \boldsymbol{I}_N - \boldsymbol{A}\boldsymbol{V}(\boldsymbol{s}_{t-1}\boldsymbol{s}'_{t-1} - \boldsymbol{I}_N)\boldsymbol{V}'\boldsymbol{A}]^2) \\ &= \frac{1}{T} \sum_{t=1}^{N} tr([\boldsymbol{s}_t\boldsymbol{s}'_t - \boldsymbol{I}_N - \boldsymbol{B}(\boldsymbol{s}_{t-1}\boldsymbol{s}'_{t-1} - \boldsymbol{I}_N)\boldsymbol{B}]^2) \\ &= S^*(\boldsymbol{B}) \end{aligned} \tag{10}$$

where $\boldsymbol{f}_t = \boldsymbol{V}\boldsymbol{s}_t$ and $\boldsymbol{s}_t = \boldsymbol{\Lambda}^{-1/2}\boldsymbol{U}'\boldsymbol{\varepsilon}_t$. The estimates $\widehat{\boldsymbol{A}}$ and $\widehat{\boldsymbol{B}}$ minimize respectively $S(\boldsymbol{A})$ and $S^*(\boldsymbol{B})$ from the first-order conditions, and $\widehat{\boldsymbol{B}} = \boldsymbol{V}'\widehat{\boldsymbol{A}}\boldsymbol{V}$. This implies that $\boldsymbol{V}$ is simply a matrix of eigenvectors of matrix $\boldsymbol{B}$ from which the linear mapping matrix $\boldsymbol{W}$ and its inverse matrix $\boldsymbol{W}^{-1}$ can be computed. The authors show in their study that the eigen-vector matrix $\widehat{\boldsymbol{V}}$ of $\widehat{\boldsymbol{B}}$ is a consistent estimator of $\boldsymbol{V}$.

***Fast Independent Component Analysis* (Fast-ICA) *estimator***

The Fast-ICA method of Broda and Paolella (2008) estimates the link matrix $\boldsymbol{W}$ by factoring it as $\boldsymbol{W} = \boldsymbol{H}^{1/2}\boldsymbol{V}$ using a two-step estimation procedure based on the *shrinkage estimators* proposed by Ledoit and Wolf (2003). The following conditional log-likelihood function is maximized to give estimate of $\boldsymbol{W}$ and coefficients of the component-GARCH:

$$L(\hat{\boldsymbol{\varepsilon}}_t|\boldsymbol{\theta},\boldsymbol{W}) = T\log_e|\boldsymbol{W}^{-1}| + \sum_{t=1}^{T}\sum_{i=1}^{N}\log_e\left(GH_{\lambda_i}(f_{i,t}|\theta_i)\right) \quad (11)$$

where $GH_{\lambda_i}(f_{i,t}|\theta_i) \equiv GH\left(f_{i,t};\ \lambda_i, \mu_i\sqrt{h_{i,t}}, \frac{\omega_i}{\sqrt{h_{i,t}}}, \frac{\alpha_i}{\sqrt{h_{i,t}}}, \frac{\beta_i}{\sqrt{h_{i,t}}}\right)$ and $\boldsymbol{\theta}$ is a vector of unknown parameters in the marginal densities (Ghalanos, 2013).

***Method-of-moment* (MM) *estimator***

Boswijk and van der Weide (2009) MM estimator is obtained via a three-step estimation procedure. The procedure involves the autocorrelation properties of the zero-mean matrix-valued processes $\boldsymbol{S}_t = \boldsymbol{s}_t\boldsymbol{s}_t' - \boldsymbol{I}_N$ and $\boldsymbol{F}_t = \boldsymbol{f}_t\boldsymbol{f}_t' - \boldsymbol{I}_N$. For the process $\boldsymbol{s}_t = \boldsymbol{V}\boldsymbol{f}_t$, the autocovariance and autocorrelation matrices satisfy:

$$\boldsymbol{V}\boldsymbol{\Gamma}_i(f)\boldsymbol{V}' = E(\boldsymbol{S}_t\boldsymbol{S}_{t-i}) = \boldsymbol{\Gamma}_i(s) \quad (12)$$

as such,

$$\boldsymbol{V}\boldsymbol{\Phi}_i(f)\boldsymbol{V}' = \boldsymbol{\Phi}_i(s) = [\boldsymbol{\Gamma}_0(s)]^{-1/2}\boldsymbol{\Gamma}_i(s)[\boldsymbol{\Gamma}_0(s)]^{-1/2} \quad (13)$$

The MM estimator $\widehat{\boldsymbol{V}}_i$ is then obtained as a matrix of eigenvectors from the symmetric matrix $\frac{1}{2}\left(\widehat{\boldsymbol{\Phi}}_i(s) + \left(\widehat{\boldsymbol{\Phi}}_i(s)\right)'\right)$, where:

$$\widehat{\boldsymbol{\Phi}}_i(s) = \left[\widehat{\boldsymbol{\Gamma}}_0(s)\right]^{-1/2}\widehat{\boldsymbol{\Gamma}}_i(s)\left[\widehat{\boldsymbol{\Gamma}}_0(s)\right]^{-1/2} \quad (14)$$

with $\hat{\boldsymbol{\Gamma}}_i(s) \equiv \frac{1}{T}\sum_{t=i+1}^{T} \boldsymbol{S}_t\boldsymbol{S}_{t-i}$ , and the standardized matrix $\left(\hat{\boldsymbol{\Gamma}}_0(s)\right)^{-1/2}$ is derived from the singular value decomposition of the covariance matrix at lag zero.

### 3.4 Forecast comparisons: Diebold-Mariano (DM) test and Model Confidence Set (MCS)

The Diebold-Mariano's test compares two competing models based on the assumption of *Equal Predictive Ability* (EPA), while the MCS approach constructs a *Set of Superior Models* (SSM) with the assumption of EPA not rejected. In this study, we will consider the multivariate *Mean Squared Error* (MSE) loss function which is defined as:

$$L_{i,t} = \frac{1}{N^2} vec\left(\widehat{\boldsymbol{H}}_{i,t} - \widetilde{\boldsymbol{H}}_t\right)' vec\left(\widehat{\boldsymbol{H}}_{i,t} - \widetilde{\boldsymbol{H}}_t\right) \quad (15)$$

where $L_{i,t}$ denotes the MSE of model $i$ at time $t$; $\widehat{\boldsymbol{H}}_{i,t}$ is the model's $h$-step-ahead forecast covariance matrix , and $\widetilde{\boldsymbol{H}}_t$ is the time $t$ true covariance matrix. The true covariance matrix is unobservable, and as such, it is usually approximated by a proxy in the literature. Caporin and McAleer (2010) suggests the outer-product of the mean forecast errors defined by: $\widetilde{\boldsymbol{H}}_t \equiv \hat{\boldsymbol{e}}_t\hat{\boldsymbol{e}}_t'$, where $\hat{\boldsymbol{e}}_t \equiv \boldsymbol{r}_t - \hat{\boldsymbol{\mu}}_t$ as a proxy for the true covariance matrix.

The Diebold-Mariano's test statistic for testing the null hypothesis $H_0: E\left(d_{ij,t}\right) = E\left(L_{i,t}\right) - E\left(L_{j,t}\right) = 0$ between two competing models $i$ and $j$ is given as:

$$t_{ij} = \frac{\bar{d}_{jl}}{\left[var\left(\bar{d}_{jl}\right)\right]^{1/2}} \xrightarrow{a} N(0,1) \quad (16)$$

where $d_{ij,t} \equiv L_{i,t} - L_{j,t}$ denotes the loss differentials between the models at time $t$; $\bar{d}_{ij} \equiv \frac{1}{h}\sum_{l=1}^{h} \bar{d}_{ij,t+l}$ , $h$ denotes the forecast horizon and $var\left(\bar{d}_{ij}\right)$ is the *Heteroscedasticity and Autocorrelation Consistent* (HAC) estimate of the asymptotic variance of $\bar{d}_{jl}$. For significant *t*-values (i.e. $H_0$ is rejected), the sign of the test statistic suggest model preference: positive (negative) values indicate a preference for the second (first) model as it is associated with smaller losses. This test only facilitates pair-wise comparison of models at a time. It does not indicate the order or rank the compared models based on their forecast performances.

The MCS procedure of Hansen et al. (2011) facilitates multiple comparison and ranking of models in the order of forecast performance. The procedure creates a set of models with statistically equivalent forecast performance, using as input all pairwise loss differentials $d_{ij,t}$; $\forall i,j = 1,2,\cdots,P$ for a given loss function, where $P$ denotes the total number of fitted models. It then starts with a set $M_0$ containing the loss series of the models to be compared, performs sequential elimination of the models by testing the null hypothesis $H_0\colon E(d_{ij,t}) = 0$ for $i > j$ and for all $i,j \in M_0$. If the null hypothesis is rejected at certain confidence level $\alpha$, the worst performing model is excluded from the set. The procedure is repeated iteratively until the null hypothesis is not rejected. Hansen et al. (2005) proposed two test statistics for testing the null hypothesis based on Eq. (16). These statistics are:

$$T_R = \max_{i,j \in M_0} |t_{ij}| \tag{17}$$

$$T_{SQ} = \sum_{i,j \in M_0,\ \ i>j} (t_{ij})^2 \tag{18}$$

If the null hypothesis is rejected, the worst model can be identified with:

$$i = \underset{i \in M_0}{\operatorname{argmax}} \sum_{j \in M_0} \frac{\bar{d}_{ij}}{var\left(\sum_{j \in M_0} \bar{d}_{ij}\right)} \tag{19}$$

## 4.0 Data Description, Analysis and Discussion of Results

### 4.1 Data description

The data set consists of daily *central* exchange rates of five foreign currencies vis-à-vis the Nigerian Naira. The exchange rates are the *Danish Kroner* $(y_{1,t})$, *Euro*$(y_{2,t})$, *Japanese Yen*$(y_{3,t})$, *British Pound Sterling* $(y_{4,t})$, and *Swiss Franc* $(y_{5,t})$, which span the period 02/01/2009 through 19/03/2015. Each exchange rate time series consists of $T = 1531$ data points, and their respective percentage arithmetic return processes are denoted by $\{r_{N,t}; N = 1,2,\cdots,5\}_{t=1}^{T}$. The data set was split into two subsets: (i) the *In-Sample* data consists of the first 1510 data points covering the sample period 02/01/2009 to 05/03/2015 used for model estimation and in-sample forecast evaluations; and (ii) *Out-of-Sample* data consists of the remaining 11 data points covering the period 06/03/2015 to 19/03/2015 which are used for model out-sample forecast evaluations. The data set is available and can be downloaded from the Central Bank of Nigeria website: *www.cenbank.org/rates/ExchRateByCurrency*. The choice of the selected exchange rates is based on the availability of data in the sampled time period, and also on the fact that the respective currencies represent some of the major countries or economic zones Nigeria has trade relationships with. The value of the $N^{th}$ exchange rate at time $t$ is denoted by $y_{N,t}$, while its corresponding percentage arithmetic-returns is defined as:

$$r_{N,t} = 100 \times \left( \frac{y_{N,t} - y_{N,t-1}}{y_{N,t-1}} \right) \tag{20}$$

for $N = 1, 2, \cdots, 5$ and $t = 1, 2, \cdots, T$. The descriptive sample statistics, normality tests and the concurrent correlation matrix of the returns are presented respectively in Tables 1 and 2.[5] As shown in Table 1, the minimum percentage arithmetic returns ranges from -32.05% for the Japanese Yen to -4.94% for the Pound Sterling. Similarly, the maximum of the returns ranges from 17.61% for the Swiss Franc to 46.67% for the Yen. The sample means and medians of the returns strictly lie in the half-open interval [0, 0.04), while the standard deviations which can be viewed as a measure of unconditional volatility of the data ranges from 0.88% for the Pound Sterling to 1.98% for the Yen. The coefficients of skewness for all of the series are different from zero (they are positively skewed), while the coefficient of excess kurtosis (*Fisher's coefficient*) are all greater than 3. The normality tests (*Shapiro-Wilks* and *Jarque-Bera*) out rightly rejects the assumptions of empirical normal distribution of the return series. Table 2 reports the unconditional correlations among the return series. The table also shows that the Euro zone currency returns are more highly correlated with one another. Even though, the concurrent correlation between the Japanese Yen and the Euro is about 56%, its concurrent correlations with the other currencies (i.e. the Kroner, Pound Sterling and the Swiss Franc) are less than 50%. The estimated concurrent correlation coefficients imply that the returns co-move in a positive direction over time.

[5] Note that aberrant and missing values occur in the data set. Where any of these occurred, they are estimated with: $\hat{y}_{N,t} = \frac{1}{2}\left(y_{N,t-1} + y_{N,t+1}\right)$ for the $N^{th}$ exchange rate time series prior to data analysis.

Table 1: Descriptive statistics of the percentage arithmetic returns

| Statistics | Danish Kroner | Euro | Japanese Yen | Pound Sterling | Swiss Franc |
|---|---|---|---|---|---|
| Minimum | -14.5652 | -14.2577 | -32.0466 | -4.9352 | -11.778 |
| Median | 0.0136 | 0.0074 | 0.0063 | 0 | 0.028 |
| Mean | 0.017 | 0.0148 | 0.0269 | 0.0325 | 0.037 |
| Std. deviation | 1.2513 | 1.06 | 1.9802 | 0.8752 | 1.0804 |
| Maximum | 18.7085 | 18.7054 | 46.6655 | 18.738 | 17.607 |
| Skewness | 3.1841 | 4.7649 | 7.8433 | 7.3159 | 4.6825 |
| Kurtosis | 86.1208 | 126.2672 | 308.5123 | 149.3389 | 103.1044 |
| **Normality Tests** | | | | | |
| Shapiro - Wilks | 0.5414 | 0.5653 | 0.2577 | 0.6559 | 0.5923 |
| (p - vale) | -2.20E-16 | -2.20E-16 | -2.20E-16 | -2.20E-16 | -2.20E-16 |
| Jarque - Bera | 476692.5 | 1057456.7 | 6099475.9 | 1439232.9 | 685124.1 |
| (p - value) | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

Table 2: Concurrent correlation matrix of the exchange rate arithmetic returns

| | Danish Kroner | Euro | Japanese Yen | Pound Sterling | Swiss Franc |
|---|---|---|---|---|---|
| Danish Kroner | 1 | | | | |
| Euro | 0.7618 | 1 | | | |
| Japanese Yen | 0.4802 | 0.5598 | 1 | | |
| Pound Streling | 0.6042 | 0.7223 | 0.3395 | 1 | |
| Swiss Franc | 0.4856 | 0.5744 | 0.3048 | 0.544 | 1 |

In order to eliminate dynamic linear dependences inherent in the data, optimal orders of VAR($p$) were determined using the `VARorder` function of R package `MTS`. The AIC and HQIC information criteria select optimal order of $p = 10$, while the BIC selects the order $p = 4$. Table A1 shows the results of the order determination. The VAR(10) mean model was estimated and its residuals diagnosed for adequacy. The multivariate Ljung-Box test shows that the fitted model captures all of the linear dependences inherent in the data adequately. Figure A1 of Appendix A shows results of these diagnostic checks. Similarly, the multivariate ARCH-LM tests on the squared residuals indicate the presence of significant ARCH effects, which implies that the conditional variances of the exchange rate returns are time-varying. Results of the ARCH test are presented in Table A2.

### 4.2 Estimation and Analysis of GO-GARCH model

The innovation vector: $\{\hat{\boldsymbol{\varepsilon}}_t = \boldsymbol{r}_t - \hat{\boldsymbol{\mu}}_t; t = 1, 2, \cdots, T\}$, where $\hat{\boldsymbol{\mu}}_t \equiv \boldsymbol{\phi} + \sum_{i=1}^{10} \hat{\boldsymbol{\Phi}}_i \boldsymbol{r}_t$ denotes the fitted VAR (10) model was employed in the estimation of the GO-GARCH models. The four estimators discussed earlier were used for fitting the GO-GARCH models. A GARCH (1, 1) structure was assumed for the latent factors.[6]

### 4.3 Diagnostics of estimated GO-GARCH models

The fitted models were assessed for goodness-of-fit using the Hosking (1980) and Li and McLeod (1981) multivariate portmanteau tests for assessing the fit of our estimated GOGARCH models. The results of the tests for each of the estimated GO-GARCH (1, 1) models are presented in Table B1. All of the fitted models are adequate at the $\alpha = 5\%$ level of significance, but the GO-GARCH model estimated by the maximum likelihood estimator appears to have a better fit to the data compared with the other estimators.

### 4.4 Forecast performance evaluations

We evaluate forecast performance of the fitted models by testing the null hypothesis of EPA using the DM test for both the in-sample and out-of-sample data sets. The results for the DM tests and their corresponding p-values are presented in Tables B2 and B3 respectively. From Table B2, it is noted that all of the estimated models are statistically equivalent at $\alpha = 10\%$ level of significance. However, the null hypothesis of EPA is rejected (at the $\alpha = 5\%$ level of significance) for the tests between the ML and NLS estimators, and NLS and MM estimators for forecast horizon $h = 1$ and $h = 5$; and also between NLS and MM estimators for $h = 30$. The results indicate preference for the GO-GARCH model estimated with the NLS estimator. The DM tests in the out-of-sample show that the models estimated with the ML, NLS and MM estimators are preferred to the model estimated with the Fast-ICA estimator.

To rank the models in the order of forecast performance in the in-sample and out-of-sample data sets, the MCS test procedure was used. The results of the in-sample and out-of-sample MCS test results are presented in Tables B4 and B5 respectively. From Table B4, it is obvious that the estimated models are

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[6] Details of the results of the fitted models are not presented here due to space and size of the outputs, but are available upon request.

statistically equivalent at $\alpha = 10\%$, $\alpha = 5\%$ and $\alpha = 1\%$ confidence levels respectively; as such none of the models was eliminated. But then, the rankings of the models in terms of their forecast performance indicate that the GO-GARCH model estimated with the NLS estimator is ranked 1st, the MM estimator ranked 2nd, the ML estimator ranked 3rd and the Fast-ICA estimator ranked 4th. In the out-of-sample test results, the set superior models created include only the GO-GARCH models estimated with the NLS and MM estimators respectively. These models are ranked 1st and 2nd respectively. Having determined the set of superior models for forecasting the system of exchange rates, the study proceed to fit two forecast models (i.e. the GOGARCH-NLS and GOGARCH-MM) using the entire data set. A brief summary of the fitted coefficients of are presented in Tables 3 and 4 respectively (Appendix B presents detailed summary of the fitted models).[7] The Hosking and Li and McLeod tests on the residuals of these models does not show any lack of fit.

The results presented in Table 3 and Table 4 show that some of the estimated component GARCH processes are highly persistent, especially component GARCH 2 and 4 for the GOGARCH-NLS model, and component GARCH 1, 3 and 4 for GOGARCH-MM model respectively. Another observation from the tables is that the estimated shock coefficients from the NLS estimator (defined by $\hat{\alpha}_2$, $\hat{\alpha}_4$ and $\hat{\alpha}_5$) decay at a faster rate compared to those of the MM estimator. Likewise, the shocks from the MM estimator (defined by $\hat{\alpha}_1$ and $\hat{\alpha}_3$) decay faster compared to those of the NLS estimator. These models provide different magnitudes of estimates of the time-varying conditional variances. Figures B1 and B2 present the time plots of the estimated conditional variances, while Fig. B3 and Fig. B4 present the time plots of the estimated pair-wise conditional correlations for the GOGARCH-NLS and GOGARCH-MM models respectively. The conditional variance time plots that the estimates of the conditional variances are higher in the NLS estimator. Both estimators however capture three regimes of high volatilities in the exchange rate return processes. The period of highest volatility occurs towards the end of the sample data; the cause of this volatility may be attributed to devaluation of the Naira. Table B6 presents the sample ranges of the estimated time-varying conditional variances for each exchange rate returns series.

[7]All of the GOGARCH models considered in this study were estimated using the R package 'gogarch' of Pfaff (2014).

The time plots of the conditional correlations also show periods of high volatilities in the co-movements of the exchange rates. For the NLS estimator, the time-varying pair-wise correlations follow decreasing trends towards the end of the sample period. The pair-wise conditional correlations between the Euro & Kroner, Pound & Euro, Swiss Franc & Euro, Pound & Yen, Swiss & Yen, and Swiss & Pound follow slightly increasing trends but also decrease towards the end of the sample period. The other pair-wise correlations fluctuate about their respective average conditional correlations. The estimated pair-wise conditional correlations for the MM estimator are similar to those of the NLS estimator, but they appear to be more volatile and more random. These pair-wise correlation coefficients also decrease towards the end of the sample except for those between the Euro & Kroner, Swiss & Yen, and Pound & Yen which appear to decrease slowly. It is worth to note that the magnitudes of the estimated pair-wise correlation coefficients are essentially the same for both the NLS and the MM estimators. These pairwise conditional correlations are reported in Table B7.

Table 3: Estimated coefficients of GOGARCH model via NLS estimator

| | | **Linear Mapping Matrix** | | | |
|---|---|---|---|---|---|
| | [,1] | [,2] | [,3] | [,4] | [,5] |
| [,1] | 0.46605 | -0.21169 | -0.8136 | -0.39821 | -0.06275 |
| [,2] | 0.22917 | 0.02223 | -0.7911 | -0.10471 | -0.4137 |
| [,3] | -0.58665 | -0.68155 | -1.05678 | -0.0131 | -0.06938 |
| [,4] | -0.03791 | 0.37736 | -0.60218 | -0.37617 | -0.06419 |
| [,5] | 0.25829 | 0.29226 | -0.81302 | 0.35659 | 0.32485 |
| | | **GARCH Coefficients** | | | |
| Component | omega | alpha | beta | | |
| 1 | 1.0855e – 01 | 0.11032 | 0.79975 | | |
| 2 | 1.0820e – 03 | 0.08559 | 0.92813 | | |
| 3 | 3.8624e – 01 | 1 | 0.14495 | | |
| 4 | 1.0007e – 06 | 0.04919 | 0.96647 | | |
| 5 | 2.0211e – 01 | 0.20548 | 0.63909 | | |

Table 4: Estimated coefficients of GOGARCH model via MM estimator

| | | | | | |
|---|---|---|---|---|---|
| | | | **Linear Mapping Matrix** | | |
| | [,1] | [,2] | [,3] | [,4] | [,5] |
| [,1] | 0.85718 | 0.4019 | 0.14043 | 0.37538 | 0.17193 |
| [,2] | 0.57261 | 0.69076 | 0.12615 | 0.13858 | -0.14422 |
| [,3] | 0.58351 | 0.27526 | 1.20138 | 0.01707 | -0.26568 |
| [,4] | 0.08942 | 0.63013 | 0.18898 | 0.42098 | 0.18462 |
| [,5] | 0.35201 | 0.63884 | 0.21769 | -0.35188 | 0.5853 |
| | | **GARCH Coefficients** | | | |
| Component | omega | alpha | beta | | |
| 1 | 0.00753 | 0.03476 | 0.96507 | | |
| 2 | 0.05507 | 1 | 0.45554 | | |
| 3 | 0.00186 | 0.09491 | 0.91804 | | |
| 4 | 0.00187 | 0.05311 | 0.96258 | | |
| 5 | 0.26801 | 0.31404 | 0.4767 | | |

## 5.0 Conclusion

The study aimed at estimating GO-GARCH models for forecasting the dynamics of a system of five exchange rates vis-à-vis the Nigerian Naira. Four estimators were considered for fitting the models, and tests for forecast performance in both in-sample and out-of-sample were conducted using the Diebold-Mariano (DM) tests and the Model Confidence Set (MCS) procedure. The results of the tests show that the fitted models are statistically equivalent in terms of in-sample forecast performance, but in the out-of-sample tests, the GO-GARCH models estimated with the NLS and MM estimators constitute the Set of Superior Models (SSM). These models were considered best for forecasting the dynamics of the system of Naira exchange rate returns within the GO-GARCH framework. The estimators were based on the assumption of multivariate normal distribution. As to whether the choice of returns distributions will improve forecast performance of the models was not considered in the study, but is left for further research.

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## Appendix A

Table A1: Selected optimal orders of VAR (p) models based on the multivariate information criteria[8]

Selected order: AIC = 10
Selected order: BIC = 4
Selected order: HQIC = 10
Summary table:

| *p* | AIC | BIC | HQIC | M(*p*) | *p* - *value* |
|---|---|---|---|---|---|
| 0 | -0.8303 | -0.8303 | -0.8303 | 0 | 0 |
| 1 | -1.3694 | -1.2823 | -1.337 | 863.7682 | 0 |
| 2 | -1.51 | -1.3357 | -1.4452 | 260.8567 | 0 |
| 3 | -1.726 | -1.4646 | -1.6287 | 373.0804 | 0 |
| 4 | -1.8505 | -1.5019 | -1.7208 | 235.0789 | 0 |
| 5 | -1.8724 | -1.4367 | -1.7103 | 81.3806 | 0 |
| 6 | -1.895 | -1.3722 | -1.7005 | 82.1574 | 0 |
| 7 | -1.8911 | -1.2811 | -1.6641 | 42.5949 | 0.0155 |
| 8 | -1.9161 | -1.2189 | -1.6566 | 85.0168 | 0 |
| 9 | -2.0268 | -1.2426 | -1.735 | 210.9399 | 0 |
| 10 | -2.3067 | -1.4352 | -1.9823 | 457.9651 | 0 |
| 11 | -2.2963 | -1.3378 | -1.9396 | 32.6606 | 0.1398 |
| 12 | -2.2825 | -1.2368 | -1.8933 | 27.3626 | 0.338 |
| 13 | -2.2735 | -1.1407 | -1.8519 | 34.4824 | 0.098 |

[8]The functions `refVAR`, `mq`and `MarchTest`of the R package `MTS` of Tsay(2014) was used in the estimation and diagnostics of the VAR models.

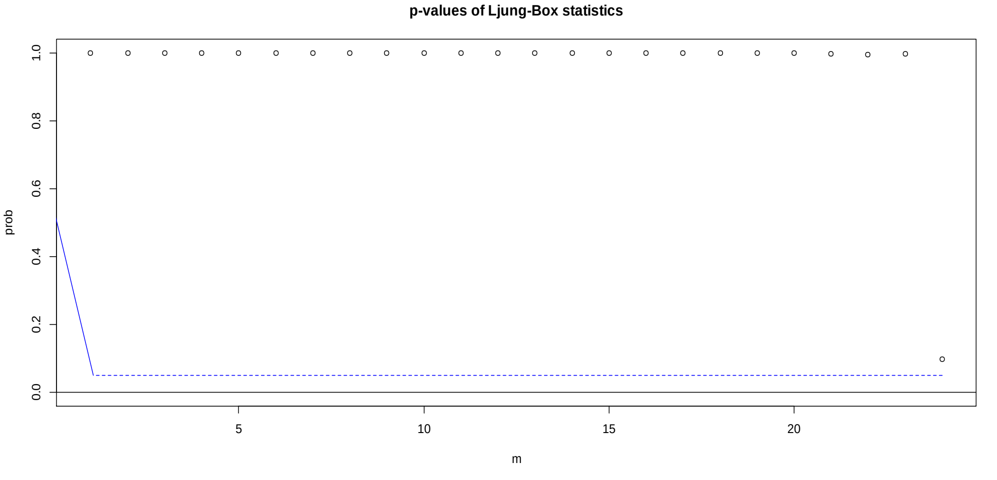


Figure A1: Diagnostic plot of multivariate portmanteau Ljung-Box test on the residuals from the fitted VAR (10) models.

Table A2: Multivariate ARCH tests on squared residuals from fitted VAR (10) model

| **Lag** | **ARCH - LM Test** | **Statistic** | **p - value** |
|---|---|---|---|
| 10 | Q(*m*) | 170.3796 | 0.0000 |
| 10 | Q*(m) | 2358.738 | 0.0000 |
| 20 | Q(*m*) | 171.7346 | 0.0000 |
| 20 | Q*(m) | 2399.078 | 0.0000 |
| 30 | Q(*m*) | 363.1023 | 0.0000 |
| 30 | Q*(m) | 6091.971 | 0.0000 |
| 40 | Q(*m*) | 364.9385 | 0.0000 |
| 40 | Q*(m) | 6153.335 | 0.0000 |

## Appendix B

Table B1: Goodness-of-fit tests on the estimated GOGARCH models[9]

| Estimator | Lag | Statistic | Hosking Test<br>Degree of freedom | p - value | Li & McLeod Test<br>Statistic | p - value |
|---|---|---|---|---|---|---|
| Maximum likelihood | 5 | 139.2904 | 125 | 0.2288 | 139.2579 | 0.2288 |
| | 10 | 250.1662 | 250 | 0.3996 | 250.155 | 0.3986 |
| | 15 | 363.6848 | 375 | 0.4745 | 363.7675 | 0.4745 |
| | 20 | 444.6371 | 500 | 0.7922 | 445.25 | 0.7912 |
| Nonlinear least squares | 5 | 193.1833 | 125 | 0.031 | 193.0506 | 0.031 |
| | 10 | 319.028 | 250 | 0.0579 | 318.8363 | 0.0569 |
| | 15 | 422.1668 | 375 | 0.1518 | 422.1666 | 0.1508 |
| | 20 | 521.0518 | 500 | 0.2637 | 521.355 | 0.2587 |
| Method of moment | 5 | 186.0793 | 125 | 0.042 | 185.9549 | 0.042 |
| | 10 | 316.192 | 250 | 0.08 | 315.9793 | 0.08 |
| | 15 | 433.2828 | 375 | 0.1269 | 433.1307 | 0.1269 |
| | 20 | 528.3005 | 500 | 0.2478 | 528.5132 | 0.2468 |
| Fast ICA | 5 | 264.2012 | 125 | 0.013 | 263.9584 | 0.013 |
| | 10 | 389.7878 | 250 | 0.017 | 389.4889 | 0.017 |
| | 15 | 470.1582 | 375 | 0.0629 | 470.2288 | 0.0619 |
| | 20 | 568.4552 | 500 | 0.1169 | 568.844 | 0.1159 |

Table B2: In-sample forecast performance evaluations – p values in parenthesis

| | | | | |
|---|---|---|---|---|
| **Test for h=1** | | | | |
| | ML | NLS | MM | Fast - ICA |
| ML | - | 2.4854 (-0.01296) | 0.9715 (0.3298) | 0.4435 (0.6574) |
| NLS | | - | -2.1165 (0.0343) | -1.0182 (0.3086) |
| MM | | | - | -0.2236 (0.823) |
| **Test for h = 5** | | | | |
| | ML | NLS | MM | Fast - ICA |
| ML | - | 2.204 (0.02756) | 0.8897 (0.3736) | 0.4226 (0.6726) |
| NLS | | - | -2.1881 (0.02869) | -1.0129 (0.3111) |
| MM | | | - | -0.2223 (0.8241) |
| **Test for h = 30** | | | | |
| | ML | NLS | MM | Fast - ICA |
| ML | - | 1.6712 (0.09473) | 0.7514 (0.4525) | 0.3581 (0.7203) |
| NLS | | - | -2.3269 (0.02) | -0.9779 (0.3282) |
| MM | | | - | -0.2186 (0.827) |

Note: p-values in parentheses

[9]The multivariate portmanteau tests were carried out with the use of the R package 'portes' developed by Mahdi et al.(2014)

Table B3: Out-of-sample forecast evaluations (EPA)

| Test for h=1 | | | | |
|---|---|---|---|---|
| | ML | NLS | MM | Fast - ICA |
| ML | - | 1.1254 (0.26659) | 1.5054 (0.1387) | -3.2108 (0.00234) |
| NLS | | - | 1.5664 (0.1237) | -3.3666 (0.0015) |
| MM | | | - | -4.3974 (5.9e-05) |

Table B4: In-sample MCS test results: the created Superior Set of Models (SSM)

| Estimator | Rank R | TSQ v_R | p - value | Rank R | TR v_R | p - value |
|---|---|---|---|---|---|---|
| | | alpha = 10% | | | alpha = 10% | |
| ML | 3 | 0.2947 | 0.7408 | 3 | 0.2933 | 0.74 |
| NLS | 1 | -0.0336 | 1 | 1 | -0.0336 | 1 |
| MM | 2 | 0.0336 | 0.9964 | 2 | 0.0336 | 0.995 |
| Fast ICA | 4 | 0.5642 | 0.3914 | 4 | 0.564 | 0.4046 |
| | | alpha = 5% | | | alpha = 5% | |
| ML | 3 | 0.2932 | 0.7372 | 3 | 0.2928 | 0.744 |
| NLS | 1 | -0.0336 | 1 | 1 | -0.0336 | 1 |
| MM | 2 | 0.0336 | 0.9958 | 2 | 0.0336 | 0.996 |
| Fast ICA | 4 | 0.5712 | 0.3968 | 4 | 0.5629 | 0.4058 |
| | | alpha = 1% | | | alpha = 1% | |
| ML | 3 | 0.2911 | 0.747 | 3 | 0.2939 | 0.7442 |
| NLS | 1 | -0.0336 | 1 | 1 | -0.0336 | 1 |
| MM | 2 | 0.0335 | 0.9968 | 2 | 0.0336 | 0.9962 |
| Fast ICA | 4 | 0.5683 | 0.3882 | 4 | 0.5627 | 0.3988 |

Table B5: Out-of-sample MCS test results: the created Superior Set of Models (SSM)

| Estimator | Rank R | TSQ v_R | p - value | Rank R | TR v_R | p - value |
|---|---|---|---|---|---|---|
| | | alpha = 10% | | | TR: alpha = 10% | |
| NLS | 1 | -0.9994 | 1 | 1 | -0.9995 | 1 |
| MM | 2 | 0.9995 | 0.1778 | 2 | 0.9994 | 0.187 |
| | | alpha = 5% | | | TR: alpha = 5% | |
| NLS | 1 | -0.9989 | 1 | 1 | -0.9997 | 1 |
| MM | 2 | 0.9999 | 0.1764 | 2 | 0.9992 | 0.1856 |
| | | alpha = 1% | | | TR: alpha = 1% | |
| NLS | 1 | -0.9995 | 1 | 1 | -0.9996 | 1 |
| MM | 2 | 0.9994 | 0.1792 | 2 | 0.9993 | 0.169 |

Table B6: Ranges of the estimated conditional variances by the NLS and MM estimators

| Estimator | Danish Kroner | Euro | Japanese Yen | Pound Sterling | Swiss Franc |
|---|---|---|---|---|---|
| NLS | 0.45 – 196.83 | 0.41 – 183.63 | 0.74 – 331.68 | 0.20 – 105.79 | 0.42 – 194.11 |
| MM | 0.26 – 35.82 | 0.16 – 98.73 | 0.26 – 76.29 | 0.09 – 82.84 | 0.28 – 99.53 |

Table B7: Averages of estimated pair-wise conditional correlations

| Exchange rate pair | NLS estimator | MM estimator |
|---|---|---|
| Danish Kroner & Euro | 0.8154 | 0.8411 |
| Danish Kroner & Japanese Yen | 0.4586 | 0.5375 |
| Danish Kroner & Pound Sterling | 0.6832 | 0.6612 |
| Danish Kroner & Swiss Franc | 0.4571 | 0.5256 |
| Euro & Japanese Yen | 0.5207 | 0.6189 |
| Euro & Pound Sterling | 0.7082 | 0.6678 |
| Euro & Swiss Franc | 0.4628 | 0.4919 |
| Pound & Japanese Yen | 0.4141 | 0.341 |
| Swiss Franc & Japanese Yen | 0.3199 | 0.2974 |
| Swiss Franc & Pound Sterling | 0.4248 | 0.4796 |

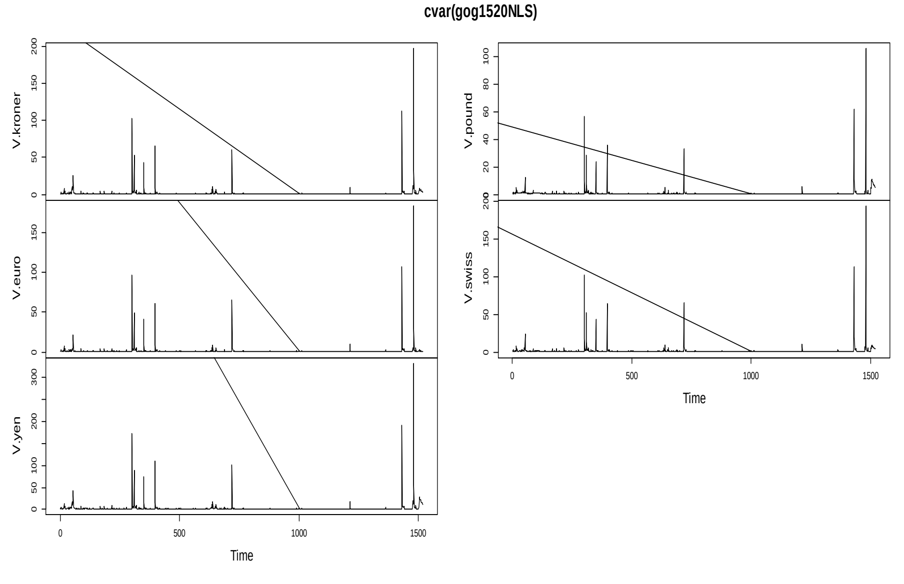


Figure B1: Time plots of conditional variances estimated with the NLS estimator.

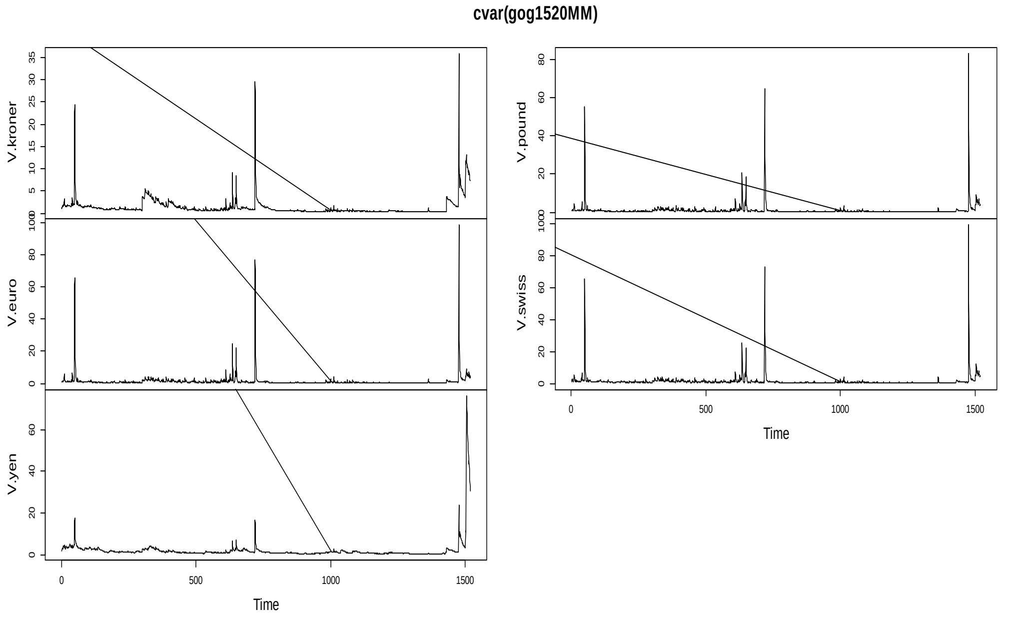


Figure B2: Time plots of conditional variances estimated with the ML estimator.

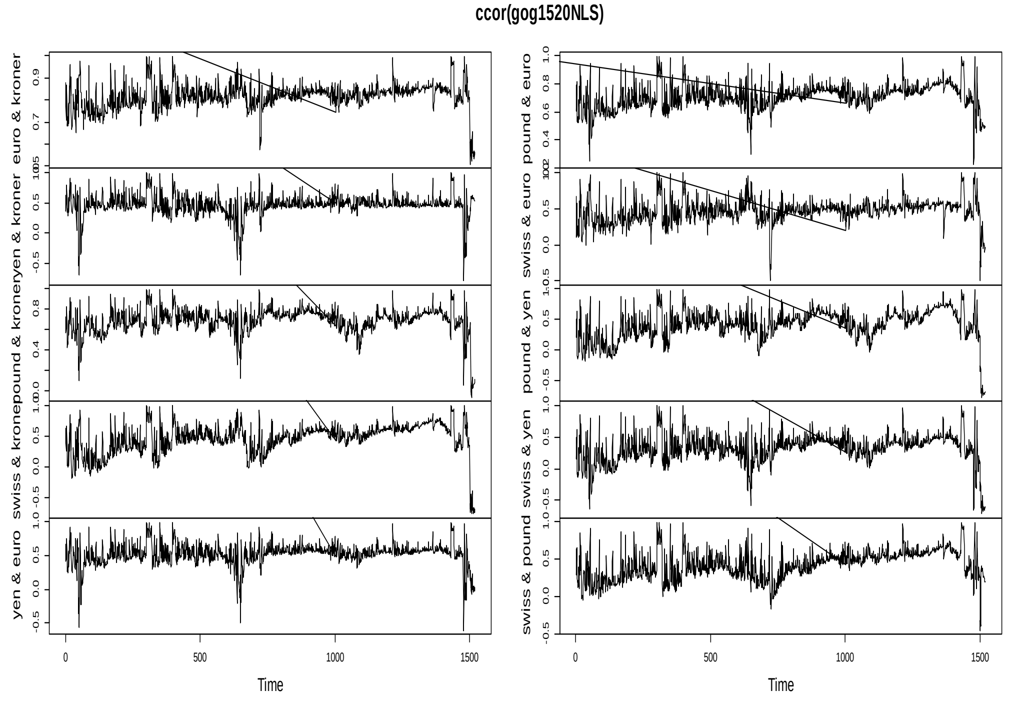

Figure B3: Time plots of conditional correlations estimated with the NLS estimator.

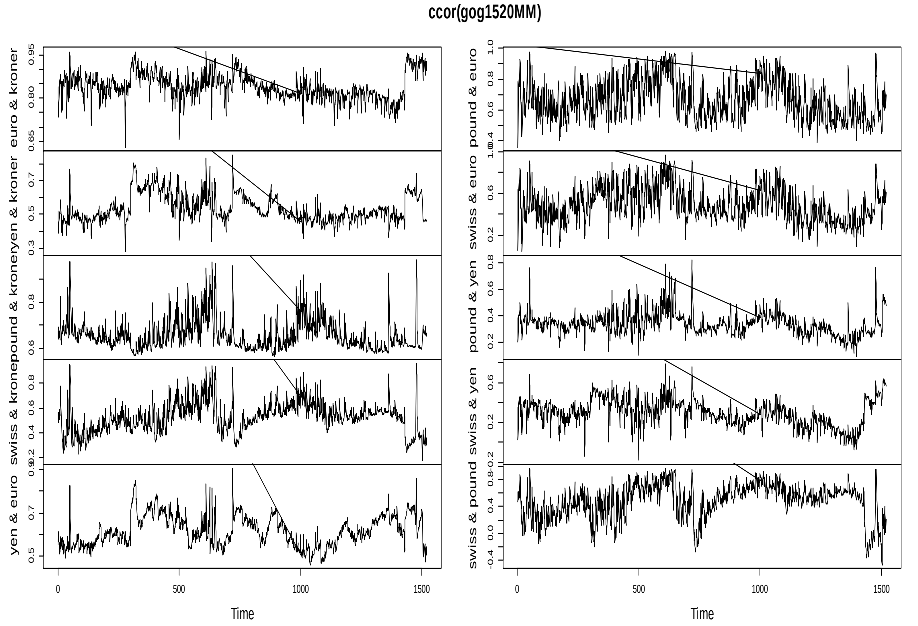


Figure B4: Time plots of conditional correlations estimated with the ML estimator.

## Appendix C

***R output of GOGARCH (1, 1) model estimated with Nonlinear Least Squares(NLS) estimator***

```
*********************************

*** Summary of GO-GARCH Model ***

*********************************

Used object: resVAR10

Components estimated by: non-linear Least-Squares

Formula for component GARCH models: ~ garch(1, 1)

The Inverse of the Linear Map Z:

            [,1]       [,2]        [,3]       [,4]       [,5]

[1,]  0.96966862  0.2862890 -0.66067452 -0.8536580  0.2421152

[2,] -0.80825129  0.2819243 -0.46145859  1.0977372  0.3212513

[3,] -0.06114769 -0.2166278 -0.29281943 -0.2421137 -0.3980627
```

```
[4,] -0.95611777  0.9540715  0.04550961 -1.1412227  0.8145626

[5,]  0.85264668 -2.0707199  0.15765528  0.3379239  0.7064122

****************************************

*** Estimated Component GARCH models ***

****************************************

Component GARCH model of y1

        Estimate  Std. Error   t value     Pr(>|t|)

omega  0.1085455  0.01937282  5.602981 2.106969e-08

alpha1 0.1103192  0.02226585  4.954636 7.246569e-07

beta1  0.7997489  0.03245385 24.642648 0.000000e+00

Component GARCH model of y2

          Estimate  Std. Error     t value  Pr(>|t|)

omega  0.001082012 0.001149684   0.9411381 0.3466341

alpha1 0.085592868 0.010067268   8.5020946 0.0000000

beta1  0.928128959 0.007903403 117.4340863 0.0000000

Component GARCH model of y3

        Estimate  Std. Error  t value    Pr(>|t|)

omega  0.3862408  0.04155187 9.295389 0.000000000

alpha1 1.0000000  0.12058007 8.293244 0.000000000

beta1  0.1449505  0.04501710 3.219899 0.001282356

Component GARCH model of y4

           Estimate  Std. Error      t value  Pr(>|t|)

omega  1.000658e-06 0.001033519 9.682049e-04 0.9992275

alpha1 4.918693e-02 0.004523904 1.087267e+01 0.0000000

beta1  9.664675e-01 0.002550307 3.789613e+02 0.0000000

Component GARCH model of y5

        Estimate  Std. Error   t value     Pr(>|t|)

omega  0.2021125  0.04200716  4.811383 1.498895e-06

alpha1 0.2054795  0.04599183  4.467740 7.905023e-06

beta1  0.6390892  0.04651573 13.739207 0.000000e+00
```

***R output of GOGARCH (1, 1) model estimated with Method-of-Moment (MM) estimator***

```
********************************

*** Summary of GO-GARCH Model ***

********************************

Used object: resVAR10

Components estimated by: Methods of Moments

Formula for component GARCH models: ~ garch(1, 1)

The Inverse of the Linear Map Z:

           [,1]       [,2]         [,3]       [,4]       [,5]

[1,]  1.0949315  0.2359173 -0.003628352 -1.0093441  0.0532223

[2,] -0.8637336  1.2016431 -0.172174134  0.6139712  0.2779993

[3,] -0.1651165 -0.7439843  0.840390967  0.4464983  0.1058159

[4,]  0.7777578 -0.7903328 -0.051498300  1.2431143 -0.8386995

[5,]  0.8132359 -1.6518964 -0.153421744  0.5181983  0.8295037

***************************************

*** Estimated Component GARCH models ***

***************************************

Component GARCH model of y1

          Estimate  Std. Error    t value     Pr(>|t|)

omega  0.007532605 0.001097025   6.866395 6.584511e-12

alpha1 0.034763166 0.005033877   6.905843 4.990675e-12

beta1  0.965068372 0.003430494 281.320503 0.000000e+00

Component GARCH model of y2

         Estimate  Std. Error   t value     Pr(>|t|)

omega  0.05507291 0.009268705  5.941813 2.818872e-09

alpha1 0.99999999 0.081375353 12.288733 0.000000e+00

beta1  0.45553837 0.031340200 14.535273 0.000000e+00

Component GARCH model of y3

          Estimate  Std. Error   t value     Pr(>|t|)

omega  0.001856764 0.001426215  1.301882 1.929569e-01
```

```
alpha1 0.094905797 0.012505648  7.589035 3.219647e-14

beta1  0.918035078 0.010606470 86.554255 0.000000e+00
```

**Component GARCH model of y4**

```
          Estimate   Std. Error    t value     Pr(>|t|)

omega  0.001869039 0.0008741639   2.138087 3.250967e-02

alpha1 0.053106309 0.0065918963   8.056302 8.881784e-16

beta1  0.962580715 0.0028507718 337.656183 0.000000e+00
```

**Component GARCH model of y5**

```
        Estimate  Std. Error  t value     Pr(>|t|)

omega  0.2680146  0.03355344 7.987692 1.332268e-15

alpha1 0.3140414  0.05223245 6.012381 1.828181e-09

beta1  0.4766946  0.04769808 9.993999 0.000000e+00
```